# Algorithm based on simulated annealing for land-use allocation 

Inés Santé-Riveira ${ }^{\text {a,b,* }}$, Marcos Boullón-Magán ${ }^{\text {a,b }}$, Rafael Crecente-Maseda ${ }^{\text {a,b }}$, David Miranda-Barrós ${ }^{\text {a,b }}$<br>${ }^{a}$ Land Laboratory, Department of Agricultural and Forestry Engineering, University of Santiago de Compostela, Spain<br>${ }^{\mathrm{b}}$ Escuela Politécnica Superior, Campus Universitario s/n, 27002 Lugo, Spain

Received 24 August 2006; received in revised form 8 February 2007; accepted 16 March 2007


#### Abstract

This article describes the use of simulated annealing for allocation of land units to a set of possible uses on, the basis of their suitability for those uses, and the compactness of the total areas allotted to the same use or kind of use, which are fixed a priori. The results obtained for the Terra Chá district of Galicia (N.W. Spain) using different objective weighting schemes are compared with each other and with those obtained for this district under the same area constraints, using hierarchical optimization, ideal point analysis, and multi-objective land allocation (MOLA) to maximize average use suitability. Inclusion of compactness in the simulated annealing objective function avoids the highly disperse allocations typical of optimizations that ignore this sub-objective.


© 2007 Elsevier Ltd. All rights reserved.
Keywords: Multicriterion land allocation; Land uses; MOLA; Hierarchical optimization; Ideal point analysis

## 1. Introduction

Rural land-use allocation is becoming increasingly complex due to the emergence of new uses, the growing multifunctionality of rural areas, and the pressures put on these areas by urban and industrial expansion. In these circumstances, landuse allocation must try to reconcile multiple conflicting interests as rationally and transparently

[^0]as possible (Carsjens and Van der Knaap, 2002), which, among other things, involves evaluating land units not only with regard to their suitability for competing uses but also with regard to such factors as contiguity among units assigned to the same use, and the compactness of the single-use land masses so created (Aerts et al., 2003; Nalle et al., 2002).

Most land-use allocation techniques consider only one use at a time; see, for example, Carver (1991), Malczewski (1996), and Pereira and Duckstein (1993). Studies distributing land simultaneously among several mutually incompatible uses include those of Aerts and Heuvelink (2002), Aerts et al. (2003), MartínezFalero et al. (1998), and Stewart et al. (2004); see also Cromley and Hanink (2003). The computational burden on computer programs for land-use allocation,
which makes exact optimization methods such as integer programming infeasible when there are more than 2000 or 3000 land units to be allocated (Aerts et al., 2003), is increased by simultaneous consideration of multiple possible uses. It is, therefore, necessary to turn to heuristic algorithms capable of achieving near-best solutions in a reasonable time (Matthews, 2001). In particular, good results have been obtained using stochastic methods such as the simulated annealing (SA) technique originally due to Kirkpatrick et al. (1983) (Aerts et al., 2003; Alier et al., 1996; Boyland et al., 2004; Nalle et al., 2002); an additional advantage of such methods is the possibility of using nonlinear objective functions with essentially no increment in computational complexity (Tarp and Helles, 1995). Studies in which SA has been applied to land-use allocation include work by Martínez-Falero et al. (1998), who allocated 10 agricultural activities using an objective function that took six considerations into account (profit, land-use transformation cost, social costs, environmental impact, total land area, and continuity); Aerts and Heuvelink (2002), who minimized development costs while maximizing spatial compactness; Sharma and Lees (2004), who compared SA with the IDRISI multiobjective land allocation facility MOLA; and Duh and Brown (2007), who endowed their SA programme with mechanisms by which auxiliary knowledge could be used to increase search efficiency.
In the work described in this paper, we applied SA to the problem of distributing given total areas of 13 crops or covers among the 182,168 cells with a size of $100 \mathrm{~m} \times 100 \mathrm{~m}$, which make up the district of Terra Chá (Galicia, N.W. Spain). We employed an objective function that took into account the suitability of each land unit for each use, the compactness of the total area assigned to each use, and the compactness of the total area assigned to each group of similar uses. We ran the algorithm with several different sets of weights applied to these three objectives, and we compared the corresponding results with each other and with those obtained when average suitability alone was maximized using hierarchical optimization (Campbell et al., 1992; Carver, 1991; Mendoza, 1997), ideal point analysis (Barredo, 1996), and MOLA (Eastman et al., 1998). In Section 2, we describe the SA algorithm in terms allowing its generalization to problems other than the specific case of Terra Chá; in Section 3, we provide details of the application of SA and the other methods to Terra Chá in this study; and, in

Section 4, we compare the various sets of results obtained. Section 5 concludes.

## 2. The general problem and the simulated annealing algorithm

Our problem is to distribute $I$ square land units, each of unit area, among $N$ different uses under the constraint that the total number allocated to each use $n$ is the given number $I_{n}$, with $\Sigma_{n} I_{n}=I$. Also given are the suitability $A_{i n}$ of each land unit $i$ for each use, and, optionally, a set of use weights $w_{n}$ that allow preferences among uses to be taken into account as well as the suitability of the land unit for those uses (see Section 2.2). We aim to obtain solutions addressing three objectives, individually or jointly: maximization of the overall $w$-weighted suitability of the land units for the uses allocated to them; maximization of the compactness (and hence minimization of the fragmentation) of the total area assigned to any particular use; and maximization of the compactness of the total area assigned to any particular group of uses, as defined by the problem solver (for example, use groups for the case of Terra Chá are defined in Section 3).

The simulated annealing algorithm, as its name suggests, emulates the behaviour of a thermodynamic system that, as a result of configurational changes subject to the Boltzmann probability distribution, finally adopts its least-energy configuration as its temperature is gradually reduced to absolute zero (Metropolis et al., 1953). When applied in non-thermodynamic contexts, energy is replaced by the objective function to be minimized or maximized, and temperature by an arbitrary parameter $T$ that is used to control the thoroughness of the search for the optimum. The basic procedure is as follows: (1) given the current configuration of the system being optimized, a trial configuration is generated by a method that includes some element of chance. (2) The value of the objective function for the trial configuration, $E_{t}$, is compared with the value of the objective function for the current configuration, $E_{c}$. If $E_{t}$ is better than $E_{c}$, the trial configuration is adopted as the current configuration for the next iteration of the procedure. If $E_{t}$ is worse than $E_{c}$, the trial configuration is adopted as the next current configuration according to the Boltzmann probability distribution; that is to say, only with probability $\mathrm{e}^{-\left(E_{t}-E_{c}\right) / T}$ (if $E$ is to be minimized) or $\mathrm{e}^{-\left(E_{t}-E_{c}\right) / T}$ (if $E$ is to be maximized). (3) For each value of $T$, the system is allowed to
explore configuration space in this way for a number of iterations (or a number of iterations resulting in a change of configuration) that, in principle, should be sufficient to ensure that, with very high probability, $E$ values are within a range that is so good that worse $E$ values are being accepted at a lower average rate than better $E$ values, so that the average value of $E$ keeps improving. The value of $T$ is then reduced (so that better $E$ values are again favoured through a heavier filtering in the Metropolis condition) and the loop starts again. (4) The algorithm terminates upon satisfaction of some appropriate stop condition such as a pre-established number of temperature reductions.

For the present application, the whole procedure is summarized in Fig. 1. In what follows, we

```
Initialize T
Number_of_Ts : = 1
Generate starting solution \(\mathrm{S}_{\mathrm{c}}\)
\(E_{\mathrm{c}}:=E\left(\mathrm{~S}_{\mathrm{c}}\right)\)
Moves_uphill : \(=0\)
Do while Number_of_Ts \(\leq\) Number_of_Ts_Limit OR
        Moves_uphill > Moves_uphill_Limit
    Moves : = 0
    Moves_uphill : = 0
    Do while Moves \(\leq\) Moves_Limit
        Generate trial solution \(\mathrm{S}_{\mathrm{t}}\)
        \(E_{\mathrm{t}}:=E\left(\mathrm{~S}_{\mathrm{t}}\right)\)
        If \(E_{\mathrm{t}} \leq E_{\mathrm{c}}\)
            \(\mathrm{S}_{\mathrm{c}}:=\mathrm{S}_{\mathrm{t}}\)
            \(E_{\mathrm{c}}:=E_{\mathrm{t}}\)
        Moves : = Moves +1
    Else
        \(\mathrm{P}:=\) Random_number_in_(0,1)
        If \(\mathrm{P}<\exp \left(-\left(E_{\mathrm{t}}-E_{\mathrm{c}}\right) / \mathrm{T}\right)\)
            \(\mathrm{S}_{\mathrm{c}}:=\mathrm{S}_{\mathrm{t}}\)
            \(E_{\mathrm{c}}:=E_{\mathrm{t}}\)
            Moves : = Moves +1
            Moves_uphill : = Moves_uphill + 1
        Endif
    Endif
    Enddo
    \(\mathrm{T}:=\mathrm{T} \times\) Cooling_constant
    Number_of_Ts : = Number_of_Ts + 1
Enddo
```

Fig. 1. Pseudo-code summary of SA procedure.
describe in greater detail its main components: the generation of trial solutions, the objective function, and the annealing schedule.

### 2.1. Generation of land-use configurations

At the beginning of the procedure a configuration is generated that satisfies the constraint on the total area of land allotted to each land use. In order to ensure satisfaction of this constraint by successive trial configurations, these latter are generated by simply exchanging the land-use allocations of a randomly selected pair of land units. This procedure furthermore facilitates calculation of the value of the objective function for the trial configuration, which will differ from the value for the current configuration by a quantity that can be determined by consideration of only the land units affected by the proposed change in configuration.

### 2.2. The objective function

As noted above, the objective function $E$ combines three distinct subobjectives: maximization of overall $w$-weighted land suitability (function $S$ ), maximization of the compactness of the total area assigned to any particular use (function $U C$ ), and maximization of the compactness of the total area assigned to any particular group of uses (function GC). These subobjectives are combined linearly:
$E(S, U C, G C)=\alpha_{1} S+\alpha_{2} U C+\alpha_{3} G C$,
where the coefficients $\alpha_{j}$ are chosen by the problem solver, subject to the condition $\Sigma_{j} \alpha_{j}=1$, so as to control the relative importance of satisfying the individual subobjectives. To facilitate this choice and enhance its transparency, the subobjective functions are all normalized to the range $[0,1]$. We also define these functions so as to make the overall problem the minimization of $E$.

Overall $w$-weighted land suitability is evaluated in the first instance as the sum
$L S=\Sigma_{i} w_{n} A_{i n}$.
The value of the subobjective function $S$ is given by the normalizing expression
$S=\left(L S_{\max }-L S\right) /\left(L S_{\max }-L S_{\text {min }}\right)$,
where $L S_{\text {max }}$ is the value of $L S$ when each land unit $i$ is assigned its maximum weighted suitability,
$\max _{n}\left(w_{n} A_{i n}\right)$, and $L S_{\min }$ is the value of $L S$ when each land unit $i$ is assigned its minimum weighted suitability, $\min _{n}\left(w_{n} A_{i n}\right)$.

Following Fischer and Church (2003), the compactness of the total areas assigned to the various land uses is evaluated in the first instance through calculation of the total length $U B$ of the boundaries of connected areas allotted to a single use (hereinafter "use patches"):

$$
U B=\Sigma_{n}^{N} \Sigma_{r n}^{R n} P_{r n}
$$

where $P_{r n}$ is the length of the boundary of the $r_{n}$ th of the $R_{n}$ use patches with use $n$. Calculation of the boundary lengths is facilitated by the fact that the land units are unit squares, which likewise facilitates identification, for normalization purposes, of the maximum and minimum possible values of $U B$ : the maximum value $U B_{\text {max }}$, which would be realized if the area $I_{n}$ allotted to each use $n$ consisted of $I_{n}$ isolated land units, is $4 I$; and the minimum, $U B_{\text {min }}$, which corresponds to the doubtless unrealizable situation in which each use occupies a single square area, is $4 \Sigma_{n}^{N} I_{n}^{1 / 2}$. The normalized subobjective function $U C$ is given by the expression

$$
U C=\left(U B-U B_{\min }\right) /\left(U B_{\max }-U B_{\min }\right)
$$

Finally, the subobjective function $G C$ is defined similarly to $U C$ in terms of the length of the boundaries of "use group patches", $G B$.

### 2.3. The annealing schedule

The annealing schedule of an SA procedure determines the thoroughness of the search for the optimum. In general, it is recommended that the initial value of $T$ ensure that about $80 \%$ of trials are successful at this stage; this value will depend on both the way in which the objective function varies with configuration, and the configuration generating scheme, and must be identified by trial and error for each problem. In this work, the number of iterations employed at each value of $T$ was approximately $25 I$ and, following Boyland et al. (2004), each reduction of $T$ was effected by multiplication by a constant factor, which was 0.98 . Annealing was halted when fewer than five trials with worse values of $E$ had been accepted during the $25 I$ iterations with the current value of $T$ and at least 300 values of $T$ had been employed.

## 3. Application to Terra Chá

The $1832 \mathrm{~km}^{2}$ of Terra Chá is distributed between a broad southern plain in which the main towns and most farming activity are located, and a hilly northern area devoted predominantly to forestry and environmental protection. Some $53 \%$ of the total area is agricultural land, and some 7700 of its approximately 47,000 inhabitants are farm workers.

The land uses listed for Terra Chá in the Galician Agricultural Statistics yearbook for 2001 were regrouped for this study on the basis of land area occupied and similarity, similar minority uses being grouped together. As a result, the following 13 crops or covers were distinguished: maize fodder, pluriannual green fodder, other fodder crops (kale, beet), meadow, pasture, wheat, other cereals (rye, oats), potatoes, other vegetables, fruit, eucalyptus, softwood, and deciduous hardwood. These 13 uses were then grouped in the following five use groups: fodder (maize, pluriannual green fodder, other fodder crops, meadow, and pasture), cereals (wheat and other cereals), intensive agricultural crops (potatoes, other vegetables, and fruit), productive forest (eucalyptus and softwood), and protective woodland (deciduous hardwood).
The suitability of each $100 \mathrm{~m} \times 100 \mathrm{~m}$ land unit for each of the above uses was taken from Santé and Crecente (2005a). The total areas to be occupied by the various uses were determined using a decision support system employing multiobjective linear programming (Santé and Crecente, 2005b). More specifically, the interactive STEP method implemented in that system was used for joint optimization of economic, social, and environmental objectives, prioritized in this order. The resulting total areas are listed in Table 1.

Also listed in Table 1 are the weights $w_{n}$ given to the various uses. These weights were obtained as if they were to be used in an analytic hierarchy decision process (Saaty, 1980), on the basis of subjective comparison of all pairs of uses with regard to their economic importance.

With the areas, use weights and suitabilities described above, SA solutions were generated for 11 different sets of subobjective weights $\alpha_{j}$ (Table 2): one in which the only objective was maximization of overall $w$-weighted land suitability (option A in Table 2), three in which relative weights of 3:1 (the weight of the first subobjective is three times higher than the weight of the second subobjective), $1: 1$, and 1:3 were given to maximization of suitability and
use area compactness (options B-D); three in which these same relative weights were given to maximization of suitability and use group area compactness (options E-G); and four in which all three

Table 1
Total areas and weights $w_{n}$ for each use $n$ in Terra Chá problem

|  | Area (ha) | Weight $w_{n}$ |
| :--- | :--- | :--- |
| Maize | 31,799 | 0.2037 |
| Wheat | 2509 | 0.0147 |
| Other cereals | 181 | 0.0070 |
| Potatoes | 2408 | 0.0108 |
| Pluriannual green fodder | 28,835 | 0.1483 |
| Other fodder crops | 3025 | 0.0208 |
| Vegetables | 15,530 | 0.0557 |
| Fruit | 264 | 0.0083 |
| Meadow | 32,473 | 0.2770 |
| Pasture | 5129 | 0.0289 |
| Eucalyptus | 8247 | 0.0401 |
| Softwood | 23,161 | 0.0773 |
| Deciduous hardwood | 28,607 | 0.1074 |

Table 2
Subobjective weighting schemes used in SA optimization to solve Terra Chá problem

| Option | $\alpha_{1}$ | $\alpha_{2}$ | $\alpha_{3}$ |
| :--- | :--- | :--- | :--- |
| A | 1 | 0 | 0 |
| B | 0.50 | 0.50 | 0 |
| C | 0.75 | 0.25 | 0 |
| D | 0.25 | 0.75 | 0 |
| E | 0.50 | 0 | 0.50 |
| F | 0.75 | 0 | 0.25 |
| G | 0.25 | 0 | 0.75 |
| H | 0.34 | 0.33 | 0.33 |
| I | 0.50 | 0.25 | 0.25 |
| J | 0.25 | 0.50 | 0.25 |
| K | 0.25 | 0.25 | 0.50 |

subobjectives were considered, with relative weights of $1: 1: 1,2: 1: 1,1: 2: 1$, and 1:1:2 (options $\mathrm{H}-\mathrm{K}$ ). In addition, solutions maximizing suitability were sought, for the same set of total areas, by hierarchical optimization (ranking uses in accordance with the $w_{n}$ values of Table 1), by ideal point analysis (with the weights $w_{n}$ of Table 1 as objective weights, and using the Euclidean distance), and by MOLA (with the weights of Table 1 and an area tolerance of 100 ha ).

All calculations were performed on a PC with 512 Mb of RAM, a 40 Gb hard disk, and an Intel Pentium processor running at 1.4 GHz .

## 4. Results and discussion

Hierarchical optimization, ideal point analysis, and MOLA only optimize land suitability, without considering the spatial distribution of land uses. This is why the characteristics of the solutions obtained for Terra Chá by these three methods were compared with the solution provided by SA when the only objective was maximization of the suitability of the land units for the uses assigned to them (see Table 3). SA offered the solution with the greatest total suitability value, about $1 \%$ better than that achieved by MOLA, but took almost 60 times longer than MOLA and, more importantly, in the SA solution the total area allotted to each use was very much more fragmented than in the MOLA solution (see also Fig. 2). Overall, when used only to maximize total suitability, SA thus appears to be inferior to MOLA, which itself tends to generate excessively fragmented solutions (Bosque and García, 2000). Hierarchical optimization achieved the least fragmentation, with about $6 \%$ fewer use patches than in the MOLA solution, but its suitability was also lower, by about $4 \%$.

Table 3
Characteristics of solutions obtained for Terra Chá problem by hierarchical optimization, ideal point analysis, MOLA, and SA when used exclusively to maximize total suitability

|  | Hierarchical optimization | Ideal point analysis | MOLA | SA (option A) |
| :--- | :--- | :--- | :--- | :--- |
| Total suitability $(L S)$ | 122,726 | 125,146 | 127,312 | 128,705 |
| Mean use patch area (ha) | 25.33 | 22.94 | 24.00 | 14.86 |
| Use patch boundary $(U B, \mathrm{~km})$ | $13,779.6$ | $14,879.6$ | $13,864.2$ | $16,184.8$ |
| Use group patch boundary $(G B, \mathrm{~km})$ | 9345.6 | $10,170.0$ | 9440.4 | $11,220.8$ |
| No. of use patches | 7352 | 8195 | 7833 | 12674 |
| Largest use patch (ha) | 19,680 | 17,548 | 17,682 | 18,511 |
| Smallest use patch (ha) | 1 | 1 | 1 | 1 |
| Run time | 5 min | 19 min | 5 min | 4 h 57 min |



Fig. 2. Solutions obtained for Terra Chá problem by (a) SA, (b) MOLA, (c) ideal point analysis (IPA), and (d) hierarchical optimization (HO) when used exclusively to maximize total suitability.

The solution afforded by ideal point analysis was inferior to the MOLA solution as regards both suitability and fragmentation. Note that, although SA achieved the best total suitability, it did not achieve the best suitability for each individual use (see Table 4).

In Fig. 2 it can be observed that the main difference between the outcomes of the four methods is the location of intensive agricultural crops, mainly vegetable and fruit crops. In the maps obtained with SA and MOLA, the entire vegetable crop area is located in the vicinity of the main village of Terra Chá, located in approximately the centre of the region. In the SA map, this crop area is concentrated to the south of the village, whereas in the MOLA map it is distributed along the main roads leading from the village. In the map provided by ideal point analysis, the vegetable crops are
distributed in the vicinity of several villages. In the map obtained with hierarchical optimization these crops are even more dispersed, with small areas in the surroundings of several villages and roads. The spatial allocation of fruit crops is similar in the maps obtained with SA and ideal point analysis, being located along the region's main highway that intersects its south-west corner, and in the results of MOLA and hierarchical optimization, where the fruit crops are located in two small regions of low suitability in the vicinity of Terra Chá. In the case of fodder crops, the SA solution is also more similar to the MOLA map, especially in the case of maize. The pluriannual fodder crops are dispersed across the maps obtained with the four methods, mainly on the hierarchical optimization map, whereas with ideal point analysis these crops are quite concentrated in the eastern part of the region, which has

Table 4
Suitabilities of individual uses obtained for Terra Chá problem by hierarchical optimization, ideal point analysis, MOLA, and SA when used exclusively to maximize normalized total suitability $S$

|  | Hierarchical optimization | Ideal point analysis | MOLA | SA $\left(\alpha_{1}=1, \alpha_{2}=0, \alpha_{3}=0\right)$ |
| :--- | :--- | :--- | :--- | :--- |
| Maize fodder | $21,240.3$ | $20,322.2$ | $21,819.6$ | $21,848.6$ |
| Wheat | 736.6 | 781.7 | 666.5 | 859.1 |
| Other cereals | 11.2 | 12.0 | 10.0 | 19.0 |
| Potato | 545.6 | 642.9 | 464.0 | 641.2 |
| Pluriannual green fodder | $16,078.3$ | $20,747.6$ | $17,051.7$ | $19,132.7$ |
| Other fodder crops | 1273.3 | 2227.2 | 948.9 | 1343.9 |
| Vegetables | 9524.3 | $11,231.3$ | $11,768.9$ | $11,006.2$ |
| Fruit | 22.0 | 59.0 | 85.0 |  |
| Meadow | $25,893.9$ | 25085.1 | 31583.0 | $24,958.9$ |
| Pasture | 3134.0 | 5472.3 | 5412.6 | 4963.0 |
| Eucalyptus | $19,269.5$ | $20,092.0$ | $20,159.4$ |  |
| Softwood | $20,047.1$ | $20,797.2$ | $20,512.9$ |  |
| Deciduous hardwood | $19,764.3$ |  |  |  |

Table 5
Total suitability $(L S)$, total use patch boundary length $(U B)$, and total use group patch boundary length $(G B)$ of SA solutions obtained for Terra Chá problem with subobjective weightings of Table 2, together with corresponding run times

| Option $\left(\alpha_{1} / \alpha_{2} / \alpha_{3}\right)$ | Total suitability $(L S)$ | Use patch boundary $(U B, \mathrm{~km})$ | Use group patch boundary $(G B, \mathrm{~km})$ | Run time |
| :--- | :--- | :--- | :--- | :--- |
| A $(1 / 0 / 0)$ | 128,705 | $16,184.8$ | $11,220.8$ | 4 h 57 min |
| B $(0.5 / 0.5 / 0)$ | 126,037 | 6073.8 | 4293.4 | 4 h 56 min |
| C $(0.75 / 0.25 / 0)$ | 127,201 | 7096.0 | 4078.2 | 4 h 56 min |
| D $(0.25 / 0.75 / 0)$ | 125,668 | 5776.6 | 4455.6 | 4 h 51 min |
| E $(0.5 / 0 / 0.5)$ | 126,162 | $11,870.2$ | 4019.6 | 4 h 54 min |
| F $(0.75 / 0 / 0.25)$ | 126,828 | 3387.0 | 4 h 53 min |  |
| G $(0.25 / 0 / 0.75)$ | 126,013 | 3405.8 | 4 h 56 min |  |
| H $(0.34 / 0.33 / 0.33)$ | 125,303 | 3590.4 | 4 h 52 min |  |
| I $(0.5 / 0.25 / 0.25)$ | 125,787 | 3516.2 | 4 h 56 min |  |
| J $(0.25 / 0.5 / 0.25)$ | 123,160 | 3859.9 | 4 h 56 min |  |
| K $(0.25 / 0.25 / 0.5)$ | 125,026 | 6249.8 |  |  |

significant livestock activity. The SA and MOLA maps provide intermediate distributions between the former two examples. In the case of meadows, the SA and MOLA maps are again quite similar, comprising the river Miño region. Hierarchical optimization provides a similar distribution, albeit more compacted, whereas the ideal point analysis map is quite different. Pasture is distributed in small areas on the four maps, mainly in the mountainous zones. In the case of forest land uses, hardwood forest is allocated in a similar way with the four methods, located mainly in areas with high slope and protected by the Nature Network. The location of the other two forest land uses is also very similar with the four methods, especially between SA and MOLA. In short, the land-use solutions provided by SA and MOLA are quite similar and differ from the
solutions of hierarchical optimization and ideal point analysis.

Interestingly, the inferiority of SA with regard to computation time was considerably less marked when the size of the problem was increased by using land units sized $20 \mathrm{~m} \times 20 \mathrm{~m}$ instead of $100 \mathrm{~m} \times 100 \mathrm{~m}$, so that the total number of land units was $4,339,725$. In this situation, SA (with an appropriate number of iterations at each temperature) took 12 h , MOLA 3.5 h , ideal point analysis 7.5 h , and hierarchical optimization 45 min .

Table 5 shows that whenever one of the compactness subobjectives was included in the SA objective function along with the suitability subobjective, the solution obtained exhibited the expected considerable decrease in $U B$-by as much as a factor of 2.8 -with respect to the option

A solution obtained optimizing for suitability alone. Solutions B-K were also more compact than any of the solutions obtained using other methods to optimize for suitability. Reducing $\alpha_{1}$ always reduced the suitability of the solution, but in no case did suitability fall as low as the value achieved when hierarchical optimization was used to optimize suitability. When only use patch compactness was included (options $\mathrm{B}-\mathrm{D}$ ), both $U B$ and $G B$ were always reduced by more than a factor of 2 , and both $U B$ and $G B$ decreased as $\alpha_{2}$ increased. This can be seen graphically in Fig. 3, where a small region of Terra Chá is presented to show how isolated pixels disappear and how larger land-use patches are created as $\alpha_{2}$ increases. By contrast, when only use group patch compactness was included (options $\mathrm{E}-\mathrm{G}), U B$ was reduced by at most a factor of 1.4 , and although $G B$ decreased with increasing $\alpha_{3}$ (see also Fig. 4), $U B$ was greater with $\alpha_{3}=0.75$ than with $\alpha_{3}=0.50$. Varying $\alpha_{2}$ with $\alpha_{3}=0$ also caused greater variation in $U B, G B$, and suitability than varying $\alpha_{3}$ with $\alpha_{2}=0$.

Comparison of solution I with solutions B and E shows that splitting the weight assigned to compactness between use compactness and use group compactness achieves, with only a small reduction in suitability, $U B$ and $G B$ values that are only
slightly greater than when all the compactness weight is assigned to $\alpha_{2}$ or $\alpha_{3}$. With respect to solution A, solution I reduces $U B$ by $61 \%$ and $G B$ by $68 \%$ in exchange for a reduction in suitability of only $2.3 \%$. Further increasing $\alpha_{2}$ and $\alpha_{3}$ at the expense of $\alpha_{1}$ (option H ) had the expected effects on compactness. This option shows that the use of SA, assigning the same weight to each objective function, provides a much better spatial distribution of land uses than hierarchical optimization, ideal point analysis, and MOLA, as well as a higher suitability value than hierarchical optimization and ideal point analysis. Comparison of the solutions obtained with $\alpha_{1}=0.25(\mathrm{D}, \mathrm{G}, \mathrm{J}$, and K$)$ confirms that sharing weight between use compactness and use group compactness achieves better values of both $U B$ and $G B$ than when all the compactness weight is assigned to either $\alpha_{2}$ or $\alpha_{3}$, albeit at the expense of suitability.

The number of subobjectives with non-zero weight in the objective function had practically no effect on run time.

## 5. Conclusions

When the area of land to be alloted to each of a number of uses is given a priori, SA is a feasible


Fig. 3. Effects of $\alpha_{2}$ in land-use patches in a small area of solutions obtained by SA with various weighting scheme options: (a) A, (b) C, (c) B , and (d) D.


Fig. 4. Solutions obtained by SA for use groups of Terra Chá problem with various weighting scheme options: (a) A, (b) F, (c) E, and (d) G.
approach to the distribution of these areas among land units on the basis of the suitability of the units for each use and the compactness of the resulting use patches and use group patches. Application of this approach to a rural area in which 13 uses belonging to five use groups were to be allotted to some 182,168 land units suggests that when only suitability is optimized, SA is superior to hierarchical optimization, ideal point analysis, and MOLA, offering solutions that have better suitability but are more fragmented than those achieved by the other methods. For problems of the size indicated above, run time of SA on a medium-range desktop computer is a matter of hours rather than minutes, but is not prohibitive. The greatest weakness of the SA approach is precisely that, to avoid a prohibitive computational burden, it relies on being fed good $a$ priori land-use areas.

The inclusion of compactness in the SA objective function allows the achievement of significantly more compact solutions at the price of a relatively small reduction in suitability. Inclusion of only use compactness in the objective function leads to greater overall improvement than inclusion of only use group compactness, but inclusion of both
achieves results that are better than with either alone. This means that a better value of use patch and use group compactness will be achieved if the compactness weight is shared between both subobjectives than if all the weight is assigned to one of them.

## References

Aerts, J.C.J.H., Heuvelink, G.B.M., 2002. Using simulated annealing for resource allocation. International Journal of Geographical Information Science 16 (6), 571-587.
Aerts, J.C.J.H., Eisinger, E., Heuvelink, G.B.M., Stewart, T., 2003. Using linear integer programming for multi-site landuse allocation. Geographical Analysis 35 (2), 148-169.
Alier, J.L., Cazorla, A., Martínez, J.E., 1996. Optimización en la Asignación Espacial de Usos del suelo: Metodología, Casos de Aplicación y Programa Informático (Optimization of Land Use Spatial Allocation, Case Studies and Software). Ministerio de Agricultura, Pesca y Alimentación, Madrid, 235pp.
Barredo, J.I., 1996. Sistemas de Información Geográfica y Evaluación Multicriterio en la Ordenación del Territorio (Geographical Information Systems and Multicriteria Evaluation in Land Use Planning). Ra-ma, Madrid, 264pp.
Bosque, J., García, R.C., 2000. El uso de los sistemas de información geográfica en la planificación territorial (The use
of geographical information systems in land planning）．Anales de Geografía de la Universidad Complutense 20，49－67．
Boyland，M．，Nelson，J．，Bunnell，F．L．，2004．Creating land allocation zones for forest management：a simulated annealing approach．Canadian Journal of Forest Research 34，1669－1682．
Campbell，J．C．，Radke，J．，Gless，J．T．，Wirtshafter，R．M．， 1992. An application of linear programming and geographic information systems：cropland allocation in Antigua．Envir－ onment and Planning A24，535－549．
Carsjens，G．J．，Van der Knaap，W．，2002．Strategic land－use allocation：dealing with spatial relationships and fragmentation of agriculture．Landscape and Urban Planning 58，171－179．
Carver，S．J．，1991．Integrating multi－criteria evaluation with geographical information systems．International Journal of Geographical Information Systems 5 （3），321－339．
Cromley，R．G．，Hanink，D．M．，2003．Scale－independent land－use allocation modeling in raster GIS．Cartography and Geo－ graphic Information Science 30 （4），343－350．
Duh，J．－D．，Brown，D．G．，2007．Knowledge－informed Pareto simulated annealing for multi－objective spatial allocation． Computers，Environment and Urban Systems 31 （3），253－281．
Eastman，J．R．，Jiang，H．，Toledano，J．，1998．Multi－criteria and multi－objective decision making for land allocation using GIS．In：Beinat，E．，Nijkamp，P．（Eds．），Multicriteria Analysis for Land－Use Management．Kluwer Academic Publishers， Dordrecht，The Netherlands，pp．227－251．
Fischer，D．T．，Church，R．L．，2003．Clustering and compactness in reserve site selection：an extension of the biodiversity management area selection model．Forest Science 49 （4）， 555－565．
Kirkpatrick，S．，Gelatt，C．D．，Vecchi，M．P．，1983．Optimization by simulated annealing．Science 4598，671－680．
Malczewski，J．，1996．A GIS－based approach to multiple criteria group decision－making．International Journal of Geographi－ cal Information Systems 10 （8），955－971．
Martínez－Falero，E．，Trueba，I．，Cazorla，A．，Alier，J．L．， 1998. Optimization of spatial allocation of agricultural activities． Journal of Agricultural Engineering Research 69 （1），1－13．
Matthews，K．，2001．Applying genetic algorithms to multi－ objective land－use planning．Ph．D．Dissertation，Macaulay Land Use Research Institute，Aberdeen，154pp．，〈http：／／ www．macaulay．ac．uk／ladss／papers／keith－thesis．pdf〉［accessed 22 August 2007］．

Mendoza，G．A．，1997．A GIS－based multicriteria approaches to land use suitability assessment and allocation．In：Proceedings of the Seventh Symposium on Systems Analysis in Forest Resources，Traverse City，USA，pp．1－7，〈http：／／www．ncrs． fs．fed．us／pubs／gtr／other／gtr－nc205／landuse．htm＞．
Metropolis，N．，Rosenbluth，A．，Rosenbluth，M．，Teller，A．， Teller，E．，1953．Equation of state calculations by fast computing machines．Journal of Chemical Physics 21 （6）， 1087－1092．
Nalle，D．J．，Arthur，J．L．，Sessions，J．，2002．Designing compact and contiguous reserve networks with a hybrid heuristic algorithm．Forest Science 48 （1），59－68．
Pereira，J．M．C．，Duckstein，L．，1993．A multiple criteria decision－ making approach to GIS－based land suitability evaluation． International Journal of Geographical Information Science 7 （5），407－424．
Saaty，T．L．，1980．The Analytic Hierarchy Process．McGraw－ Hill，New York，287pp．
Santé，I．，Crecente，R．，2005a．Evaluación de métodos para la obtención de mapas continuos de aptitud para usos agrofor－ estales（Evaluation of methods to obtain continuous land suitability maps for agroforestry land uses）．GeoFocus 5， 40－68．
Santé，I．，Crecente，R．，2005b．Decision support system for agroforestry land use modelling．In：Proceedings of the Fifth Conference of the European Federation for Information Technology in Agriculture and Third World Congress on Computers in Agriculture and Natural Resources，Vila Real， Portugal，pp．142－149．
Sharma，S．K．，Lees，B．G．，2004．A comparison of simulated annealing and GIS based MOLA for solving the problem of multi－objective land use assessment and allocation． In：Proceedings of the 17th International Conference on Multiple Criteria Decision Analysis，Whistler，Canada，Open－ File Report AP58，〈http：／／www．bus．sfu．ca／events／mcdm／ MCDMProgram／Abstract1／AA58\％20CF\％20 Sharma\％20Land\％20Assessment．pdf $>$ ．
Stewart，T．J．，Janssen，R．，van Herwijnen，M．，2004．A genetic algorithm approach to multiobjective land use planning． Computers \＆Operations Research 31，2293－2313．
Tarp，P．，Helles，F．，1995．Multi－criteria decision－making in forest management planning－an overview．Journal of Forest Economics 1 （3），273－306．


[^0]:    *Corresponding author. Land Laboratory, Department of Agricultural and Forestry Engineering, University of Santiago de Compostela, Spain. Tel.: + $34982252231 \times 23642$; fax: +34982285926 .

    E-mail addresses: isante@lugo.usc.es (I. Santé-Riveira), marcos@dec.usc.es (M. Boullón-Magán), rcrecente@lugo.usc.es (R. Crecente-Maseda), dmiranda@lugo.usc.es
    (D. Miranda-Barrós).

